

## Polar Coordinates

Read the instructions carefully, follow the directions, and THINK! Part of the BC Curriculum includes a unit on Calculus with Polar Coordinates – however it assumes a background knowledge of the Polar Coordinate system that is typically taught in Precalculus.

By completing the following questions, you should be able to determine the similarities between the rectangular coordinate system and polar coordinate system. You should also be able to convert the coordinates of a point from one system to another.

### **Part 1: The Rectangular Coordinate System**

The points in a rectangular coordinate system are listed in the form  $(x, y)$ .

Step 1: Draw a reference triangle with the point  $(6, 8)$  on the terminal side. (Hint: We're talking trigonometry – remember back to Honors Precalculus)

1. The distance between the origin and the point  $(6, 8)$  is the hypotenuse of the triangle. How can the distance between the two points be found?
2. Find the distance: \_\_\_\_\_
3. In the unit circle, the distance is called  $r$ . Recall that the equation of a circle is  $x^2 + y^2 = r^2$ . In terms of  $x$  and  $y$ , determine a formula to find  $r$ .

$$r = \underline{\hspace{2cm}}$$

Step 2: Draw a reference triangle with the point  $(5, 5)$  on the terminal side.

1. The hypotenuse and positive  $x$ -axis form an angle. Using trigonometry, how can the measure of the angle be found?
2. Find the measure of this angle in radians.
3. Determine a formula to find the measure of the angle  $\theta$  to any point  $(x, y)$ .
4. How could the angle whose terminal side contains the point  $(5, -5)$  be found?

### **Part 2: The Polar Coordinate System**

The points in a polar coordinate system are listed in the form  $(r, \theta)$

Step 1: Draw a reference triangle with the point  $(-1, \sqrt{3})$  on the terminal side.

1. How can  $r$  be found using the rectangular coordinates?

$$r = \underline{\hspace{2cm}}$$

2. How can  $\theta$  be found using the rectangular coordinates?

$$\theta = \underline{\hspace{2cm}}$$

3. What are the polar coordinates for this point?  $\underline{\hspace{2cm}}$

4. Determine a formula to convert any point in rectangular form  $(x, y)$  to polar form  $(r, \theta)$

5. Convert the following points to polar coordinates. Give your answers in radians.

a.  $(2, 5)$

b.  $(3, -1)$

c.  $(0, 0)$

d.  $(-1, -1)$

e.  $(-5, 0)$

f.  $(4, 4\sqrt{3})$

Recall from Honors Precalculus that  $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{y}{r}$  and  $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{x}{r}$

6. Using the information above, find a formula for  $x$  and  $y$ :

$$x = \underline{\hspace{2cm}} \qquad y = \underline{\hspace{2cm}}$$

7. Using the formula above, find the rectangular coordinates for a point located 12 units away from the origin on the terminal side of an angle whose measure is  $\frac{\pi}{6}$

8. Convert each of these polar coordinates  $(r, \theta)$  to rectangular form  $(x, y)$

a.  $\left(8, \frac{\pi}{6}\right)$

b.  $\left(10, -\frac{5\pi}{4}\right)$

c.  $(3, 0)$

d.  $\left(\sqrt{2}, \frac{5\pi}{3}\right)$

9. For each of the points in part 8, give 2 other polar representations of the point. Hint: How else can you write the angle???? (STILL IN RADIANS).

### **Part 3: Polar Equations**

Use the conversions found previously to change the rectangular equations to polar form. Solve each equation for  $r$ .

1.  $x = 8$

2.  $x^2 + y^2 = 16$

3.  $y = -2$

4.  $2x + 3y = 6$

Use the conversions found previously to change these polar equations to rectangular equations.

5.  $r = \frac{5}{3}$

6.  $r \cos \theta = 7$

7.  $\theta = \frac{\pi}{3}$

8.  $r \sin \theta = 3$

**Delta Math:**

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